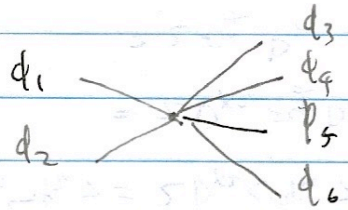
Schwartz 7.2

$$\mathcal{L} = -\frac{1}{2}\phi\Box\phi + \frac{g}{3!}\phi^3 + \frac{1}{6!}\lambda\phi^6$$

The 1st 6-pt vertex happens at 1-st order of $\frac{1}{6!}\lambda\phi^6$




$$\langle 0| T\{\phi_1\phi_2\phi_3\phi_4\phi_5\phi_6 \exp[i\int \mathcal{L}_{int}[\phi_0]]\}|0\rangle.$$

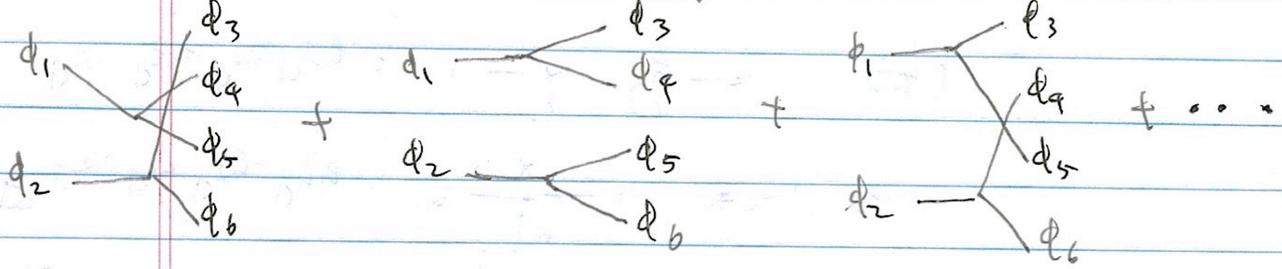
At 1st order, it's

$$\frac{i\lambda}{6!}\int d^4x \langle 0| T\{\phi_1\phi_2\phi_3\phi_4\phi_5\phi_6\ \phi_x^6\}|0\rangle$$

1 2 3 4 5 6 x x x x x x     6! symmetry factor

$$= i\lambda\int d^4x\, D_{1x}D_{2x}D_{3x}D_{4x}D_{5x}D_{6x}$$

Now we consider the diagrams




These come from 2nd order expansion of $\exp[i\int \mathcal{L}_{int}[\phi_0]]$

$$= \frac{(i)^2}{2!}\left(\frac{g}{3!}\right)^2 \int d^4x\, d^4x' \langle 0| T\{\phi_1\phi_2\phi_3\phi_4\phi_5\phi_6\, \phi_x\phi_x\phi_x\, \phi_{x'}\phi_{x'}\phi_{x'}\}|0\rangle .$$

1 2 3 4 5 6 x x x x' x' x'

$6 \times 4 \times 3 \times 3 \times 2 . =$

$$= \frac{(i)^2\, g^2 \times \cancel{6}\times\cancel{4}\times 3\times\cancel{3}\times\cancel{2}}{\cancel{2!}\,\cancel{3!}\,\cancel{3!}} \int d^4x\, d^4x'\, D_{1x} D_{3x} D_{4x} D_{2x'} D_{5x'} D_{6x'}$$

$$= 6(i^2)\, g^2 \int d^4x\, d^4x'\, D_{1x} D_{3x} D_{4x} D_{2x'} D_{5x'} D_{6x'}$$

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